International Journal of Advance and Innovative Research
Volume 12, Issue 2 (XXII): April - June 2025

ISSN 2394 - 7780

ANALYSIS OF A SINGLE-SERVER QUEUEING SYSTEM WITH DELAYED SERVER VACATIONS

[1]Mr. Jay Prakash and [2]Dr. Umesh Sharma
[1]Research Scholar, Mathematics Department in Sanskriti University, Mathura (U.P), India
[2]Associate Professor, Mathematics Department in Sanskriti University, Mathura (U.P)

ABSTRACT
This study presents a comprehensive analysis of a single-server queueing system incorporating delayed server vacations, a scenario commonly encountered in real-world service systems such as manufacturing lines, computer networks, and customer service centers. In the proposed model, the server does not immediately take a vacation upon becoming idle but instead waits for a predetermined delay period. If no new arrival occurs within this delay, the server initiates a vacation, during which it becomes unavailable for service. We develop a stochastic model to analyze the system's behavior under steady-state conditions. Key performance metrics, such as average queue length, server utilization, expected waiting time, and probability of server vacation, are derived using probability generating functions and supplementary variable techniques. The impact of the delay period and vacation parameters on system performance is also examined through numerical examples and sensitivity analysis. The findings provide insights into optimizing server utilization and improving service efficiency in systems where delayed vacations are practical or necessary. This work contributes to the broader understanding of queueing models with vacation policies and offers valuable implications for designing efficient service mechanisms.

Keywords: *Queueing theory, Single-server system, Delayed vacation, Server vacation policy.*

INTRODUCTION
The fundamental framework of this study is a queueing system featuring a single server that follows a delayed vacation policy. Implementing such a vacation strategy can enhance overall system efficiency by ensuring timely service delivery to customers with minimal delays. In this model, customer arrivals are assumed to follow a Poisson process with rate parameter λ, while service is provided in batches, with the service times following an exponential distribution characterized by parameter μ.

Model Description
This study examines an M/M (a d, b)/(2, 1) queueing system characterized by a single server with delayed vacation behavior. In this model, late arrivals are allowed to join an ongoing service batch, provided the total number of units in that batch does not exceed the predefined maximum capacity, denoted by 'd'. The server is permitted to take only one vacation at a time, and the duration of each vacation follows an exponential distribution with rate parameter θ. All other assumptions remain consistent with the standard model, except as modified by the conditions outlined in the following statement.

At each service completion or vacation return point, if a server finds n units in the queue, where $a \le n \le d - 1$, it initiates service for the entire batch. During the ongoing service, any new arrivals are allowed to join the batch until either the service is completed or the batch reaches the maximum allowable size d, whichever occurs first.

Mathematical Formulation
The queueing system can be formulated as a continuous time parameter Markov chain with states are $P_{jn}(n\geq 0,\ j = 0,1,2,3)$ and Q_{jn} $((0 \le n \le a\text{-}2),\ j = 1,2,3)$ which denotes the steady state probabilities, where 'n' represents the number of customers in the queue and 'j' signifies thestates of the server.

The states of the process are as follows

P_{0n} – the probability that one server is idle and the other on vacation,

P_{1n} – the probability that one server is busy and the other on vacation,

P_{2n} – the probability that both the servers are busy,

P_{3a-1} – the probability that one server is busy and the other server is on switchover period

Q_{1n} – the probability that one server is busy and the other server is idle,

Q_{2n} – the probability that one server is vacation and other server in on switch over period

Q_{3n} – the probability that both the servers are busy with accessible batches

The limiting probabilities corresponding to different states are

$$\boldsymbol{P_{0n} = \lim_{n\to\infty} P_{0n}(t)\,, P_{1n}(t) = \lim_{n\to\infty} P_{1n}(t)\ and\ P_{2n}(t) = \lim_{n\to\infty} P_{2n}(t)}$$

$Q_{1n}(t) = \lim_{n\to\infty} Q_{1n}(t)\,, Q_{2n}(t) = \lim_{n\to\infty} Q_{2n}(t)$ exists.

Steady State Equations

The steady state equations satisfied by P_{jn} $(j=0,1,2,3)$ and $Q_{jn}\,(j=1,2,3)$ are given by

$(\lambda + \mu)P_{00} = \mu P_{10} + \mu Q_{10}$ ……………………………………………………….(1)

$(\lambda + \theta)P_{0n} = \lambda P_{0n-1} + \mu P_{1n} + \mu Q_1 (1 \le n \le a-2)$………………………………………………..(2)

$(\lambda + \theta)P_{0a-1} = \lambda P_{0a-2} + \mu P_{1a-1} + {}_{3a-1} + \mu Q_{1a-1}$ …………………………………………………(3)

$(\lambda + \mu + \theta)P_{10} = \lambda P_{0a-1} + 2\,\mu P_{20} + \lambda Q_{3a} + \mu \sum_{n=a}^{b} P_{1n}$……… ……………….……..(4)

$(\lambda + \mu + \theta)P_{1n} = \lambda P_{1n-1} + 2\,\mu P_2 (1 \le n \le a-2)$…………………………..………(5)

$(\lambda + \mu + 2\theta)P_{1a-1} = \lambda P_{1a-2} + \alpha P_{3a-1}$ ………………………………………….……….(6)

$(\lambda + \mu + \theta)P_{1n} = \lambda P_{1n-1} + \mu P_{1n+}(n \ge a)$ ……………………………………………..(7)

$(\lambda + 2\,\mu)P_{20} = \lambda P_{3a-1} + \theta \sum_{n=d}^{b} P_{1n} + 2\mu \sum_{n=d}^{b} P_{2n} + \lambda Q_{0n-1}$ …………………….………(8)

$(\lambda + 2\,\mu)P_{2n} = \lambda P_{2n-1} + \theta\ P_{1n+b} + 2\mu P_{2n+}(n \ge 1)$………………………..…………(9)

$(\lambda + \mu + \alpha)P_{3a-1} = 2\mu P_{2a-1}(n = a-1)$……………………..………………..…………(10)

$\lambda\, Q_{2a-1} = \theta P_{0a-1} + \lambda Q_{3a-1} + 2\mu \sum_{n=d}^{d-1} Q_{3n} + \lambda Q_{2a-2}$ ……………………….……..(11)

$(\lambda + 2\,\mu)Q_{3n} = \lambda Q_{3n-1} + \theta\ P_{1n} + 2\mu P_2 (a \le n \le d-1)$…………………….……(12)

$\lambda\, Q_{20} = \theta P_{00}$ ………………….……………………………………………….……..(13)

$\lambda\, Q_{2n} = \theta P_{0n} + \lambda Q_{2n-1}(1 \le n \le a-2)$…………………….…………..……..(14)

$(\lambda + \mu)Q_{1n} = \lambda\, Q_{1n-1} + \theta P_1 (1 \le n \le a-1)$……………………………………(15)

$(\lambda + \mu)Q_{10} = \theta P_{10} + \lambda\, Q_{2a-1}$ ………………………….……………………(16)

Computation of steady state solutions

Let E denote the forward shifting operator defined by $E(P_{1n}) = P_{1n+1}$. From equation (7) we have, $(\mu\, E^{b+1} - (\lambda + \mu + \theta)E + \lambda)\, P_{1n} = 0\ (n \ge a)$. The characteristic equation of the above equation has only one real root inside the circle |Z| =1 by Rouche's theorem when $\rho =\ \lambda + \theta$ is less than 1 then

$P_{1n} = r^{n-a+1} P_{1a-1}(n \ge a-1)$ ………………………………………………… (17)

from equation (9), $(2\mu E^{b+1} - (\lambda + 2\mu)E + \lambda)\, P_{2n} = -\,\theta\ P_{1n+b+1}$ the characteristic equation of this equation has only one real root by Rouche's theorem which lies in the interval (0,1) when and using equation (15), after simplification,

$P_{2n} = (A_1 r_1^n + B r_0^n) P_{1a-1}(n \ge 0)$ …………………………………………………..(18)

Where A_1 is a constant and $B = \frac{-\theta r_0^{b-a+2}}{(\lambda+2\theta)r_0 - \lambda}$

From equation (5), substituting $n = a\text{-}2, a\text{-}3, \ldots, 1$ and solving recursively using (17) and (18),

$P_{1n} = (A_1(r_1) + BC_n(r_0))\, P_{1a-1}(1 \le n \le a-2)$………………………………………(19)

where $C_n\,(x) = \frac{2\mu R}{\lambda(x-R)}(x^n - \left(\frac{x}{R}\right)^{a-1} R^n)$, $R = \frac{\lambda}{\lambda+\mu+\theta}$ and $B = \frac{-\theta r_0^{b-a+2}}{(\lambda+2\theta)r_0 - \lambda}$

Similarly solving equation (17) recursively using (19)

$Q_{1n} = ({}_2 r^n + A_1 D_n(r_1) + BD_n(r_0))\ P_{1a-1}$, $(1 \le n \le a-1)$………………………..(20)

here A_2 is a constant, $r_2 = \frac{\lambda}{\lambda+\mu}$

$D_n\,(x) = \frac{2\mu R}{\lambda(x-R)}(\frac{x^{n+1}}{(\lambda+\mu)\mu-\lambda} - \left(\frac{x}{R}\right)^{a-1} \frac{R^n}{\theta})$, $R = \frac{\lambda}{\lambda+\mu+\theta}$ and $B = \frac{-\theta r_0^{b-a+2}}{(\lambda+2\theta)r_0 - \lambda}$

By adding (2) , (14) and using the equations (1) and (13)

$P_{0n} + Q_{2n} = \frac{\mu}{\lambda}\sum_{k=0}^{n}(P_{1n} + Q_{1n}), (0 \le n \le a-2)$

From equations (19) and (20) substituting the values of P_{1n} and Q_{1n}

$P_{0n} + Q_{2n} = \frac{\mu}{\lambda}\sum_{k=0}^{n}(A_2 r_2^n + A_1[C_n(r_1) + D_n(r_1)] + B[C_n(r_1) + D_n(r_1)])\ P_{1a-1}$ ....(21)

After simplification

$C_n(x) + D_n(x) = F_n(x) = \frac{2\mu x^{n+1}}{(\lambda+\mu)(x-\lambda)}$

Further simplifying the above equation,

$P_{0n} + Q_{2n} = \frac{\mu}{\lambda}[A_2\ \frac{1-r_2^{n+1}}{1-r_2}] + A_1 F_1(r_1)\ \frac{1-r_1^{n+1}}{1-r_1}] + F_n\ (r_0)\frac{1-r_0^{n+1}}{1-r_0}]\ P_{1a-1}$

Where $F_n(x) = \frac{2\mu x^{n+1}}{(\lambda+\mu)(x-\lambda)}$

The probability of one of the server is busy and the other server is onswitchover period can be solved by using (10)

$P_{3\,a-1}\ [A_1(r_1) + BD_n(r_0)\ )]\ P_{1a-1}$ here $G(x) = \frac{2\mu x^{a+1}}{(\lambda+\mu+\alpha)}$ ....(22)

Using the above results

$P_{1n} + Q_n = (A_2 r_2^n + A_1 F_n(r_1) r_1^n + F_n(r_0) r_0^n) P_{1a-1} (0 \le n \le a-1)$ ………..(23)

To find the probability that the servers busy and other server is busy with accessible batch limits, from equation (12)

Q_{3n}=$(A_3 r^n + AH_n(r_1) + BH_n(r_0) + k\ r^{n-a+2}) P_{1a-1} (a-1 \le n \le d)$………………(24)

Where $H_n\ (x) = \frac{2\mu R_1}{\lambda(x-R_1)}(x^{n+1} - \left(\frac{x}{R_1}\right)^a R_1^{n+1})$, r_3=$\frac{\lambda}{\lambda+2\mu}$ and k= $\frac{\theta}{(\lambda+2\mu)r_0-\lambda}$

To find the value of constants, using the results of $P_{3\,a-1}$, P_{2n} ,$_{1n}$, and Q_{1n} in equation (8)

$(\lambda + 2\mu)(A_1 + B) = [A_1(G(r_1) + D_n(r_1)) + B(G(r_0) + D_n(r_0))] + \theta r_0^{-a+1}\left(\frac{r_0^a - r_0^{d+1}}{1-r_0}\right) + 2\mu[A_1\left(\frac{r_1^a - r_1^{b+1}}{1-r_1}\right) + \mathrm{k}\left(\frac{r_0^a - r_0^{b+1}}{1-r_0}\right)] + \lambda[(A_2 r_2^n)$

By simplifying, the value of constant A_2 is obtained as follows

$A_2 = \frac{1}{r_2^{a-1}}[K_1 S(r_1) + BS(r_0) - \frac{\theta r_0}{\lambda}\left(\frac{1-r_0^{b-a+1}}{1-r_0}\right)]$ ……………………………………(25)

Where $K(x) = \frac{\lambda+2\mu}{\lambda(1-x)} - (\frac{2\mu}{\lambda})\left(\frac{x^a - x^{b+1}}{1-x}\right) - \frac{2\mu x^{a-1}}{\lambda+\mu+\alpha}$

Also to obtain the value of A_1, by adding (4) and (16),

$A_1 = \frac{1}{z(r_1)}[\ \frac{\theta r_0^a}{\lambda}\left(\frac{1-r_0^{d-a+1}}{1-r_0}\right)] - D_n(r_0)[\mu\left(\frac{r_0 - r_0^a}{1-r_0}\right) - \lambda] + \left(\frac{r_0 - r_0^{b-a+2}}{1-r_0}\right)]$…………………(26)

Where $z(r_1)$=$[F(r_1)\left\{\lambda + \mu\left(\frac{r_1^a - r_1}{1-r_1}\right)\right\} - 2\mu - \frac{S(r_1)}{r_2^{a-2}}\ ]$

By using the equation (3), the value of A is obtained as

$$A = \frac{(\theta + \mu r_0^{a-1} + 2\mu B)\left(\frac{r_0^d - r_0^{b+1}}{1-r_0}\right) - (\lambda+2\mu)B}{(\lambda+2\mu) + 2\mu\left(\frac{r_1^d - r_1^{b+1}}{1-r_1}\right)}$$ ………………………………………… ……(27)

To get the value of A_3, using (11)

$A_3 = A\ (r_1) + B\ (r_0) + r_2 Q_1(r_0)$ …………………………………………………...(28)

Where $\mathrm{Q}(x) = (1 - \frac{1}{1-r_2} + \frac{x^a - x^{d+1}}{(1-x)})$ and $\mathrm{Q}_1\ (r_0) = 1 - \frac{r_0^{d-a}}{\theta+\mu}(\theta r_0 + \mu)$

Thus, all steady-state probabilities have been expressed in terms of P_{1a-1}, which can be calculated using the normalization condition. As a result, all probabilities are fully determined by the parameters of the queueing system.

To obtain the value of P_{1a-1}, by using the normalizing condition

$\sum_{n=0}^{a-1}(P_{1n}+Q_{1n}+P_0+Q_{2n})+\sum_{n=a}^{\infty}P_{1n}+\sum_{n=0}^{\infty}P_{2n}+\sum_{n=a}^{d-1}Q_{3n}+(a-1)P_{3a-1}=1$.(29)

Substitute the results from the equations (17), (18), (21), (22) and (24)

$P_{1\,a-1}^{-1}=\quad A_3[N\quad (r_3)+\left(\frac{r_3^a-r_3^{d-1}}{1-r_3}\right)]-A_2(N+(r_2)\quad +\frac{1-r_2^a}{1-r_2}]+A_1F_n\quad (r_1)[N\quad (r_1)+\frac{1-r_1^a}{1-r_1}]\quad +$
$B\left(\frac{r_0+k}{1-r_0}\right)+A_1F_n\,(r_1)\;[N\,(r_0)\frac{1-r_0^a}{1-r_0}]+B(\frac{1}{1-r_0}+G(r_0)+kr_0^2(\frac{1-r_2^{d-a-1}}{1-r_0})]$ ....(30)

Where $G(x)=\frac{\mu}{\lambda}[\frac{a}{1-x}-\frac{x(1-x^a)}{(1-x)^2}]$

Performance Measures
Performance indicators are essential for analyzing and predicting the behavior of the system. The effectiveness of the queueing system can be demonstrated by evaluating its key performance metrics. Since the steady-state probabilities are determined, various characteristics of the queue—such as average queue length, waiting time, and server utilization—can be readily computed.

Mean Queue Length
Let L_q be the expected number of customers in the queue then

$L_q\;=\;\sum_{n=0}^{a-1}n(P_{1n}+Q_{1n})\;+\;\sum_{n=0}^{a-1}n(P_{0n}+Q_{2n})\;+\;\sum_{n=0}^{\infty}nP_{1n}\;+\sum_{n=0}^{\infty}nP_{2n}\;+\;\sum_{n=0}^{d-1}nQ_{3n}+(a-1)P_{3\,a-1})$....(31)

Using the above results, the expected number of customers in queue is

$L_q=[\,A_3\{\frac{r_1}{(1-r_1)^2}-\frac{r_1^d-r_1^{a+1}}{1-r_1}\}+A_2W_1(r_2)+A_1F_n(r_1)W_1(r_1)+BF_n(r_0)W_1(r_0)+$

$\frac{r_0}{1-r_0}\{a+\frac{r_0}{1-r_0}\}+\frac{A_1r_1}{(1-r_1)^2}+\frac{B_1r_1}{(1-r_1)^2}+k\,r_0^{n-a+2}+A_1(r_1)+BG(r_0)+A\{\frac{r_1}{(1-r_1)^2}-\frac{r_1^d-r_1^{a+1}}{1-r_1}\}$

$+\{\frac{r_0}{(1-r_0)^2}+\frac{\mu}{\lambda}(d(d-1)-a(a-1))\}]$ ....(32)

Where $W_1(x)=\frac{\mu a(a-1)}{2\lambda(1-x)}+[\frac{x(1-x^a)}{(1-x)^a}](1-\frac{\mu x}{(1-x)\lambda})$

Probability that both servers are busy (P_{2B})
The system has more than 'b' customers, then both the serverswill busy. Let P_{2B} be the probability that both the servers are busy, then

$P_{2B}=(A_1\frac{1}{1-r_1}+B\frac{1}{1-r_0})\,P_{1\,a-1}$..(33)

Probability that one server is busy and the other server is on vacation (P_{1B})
If the number of customers in the system is fewer than $a-1$, and one server is occupied with a batch within the accessible limit, the other server will remain on vacation until a minimum batch size of customers is available. Let P_{1B} denote the probability that one server is actively serving while the other is on vacation.

$P_{1B}=\left(A_2\frac{1-r_2^a}{1-r_2}+\;A_1C_n(r_1)\frac{1-r_1^a}{1-r_1}+BC_n(r_0)\frac{1-r_0^a}{1-r_0}\right)P_{1\,a-1}$..................................(34)

Probability that one server is idle the other is on vacation (P_{0B})
When the number of customers in the system is fewer than a, one of the servers remains idle while the other departs for a vacation. Let $P_0{}^B$ represent the probability that one server is idle and the other is on vacation. Then,

$P_{0n}=\frac{\mu}{\lambda}[A_2\{\frac{1}{1-r_2}-\frac{r_2(1-r_2^a)}{(1-r_2)^2}]+A_1C_1(r_1)\{\frac{r_1(1-r_1^a)}{(1-r_2)^2}\}+BC_n\,(r_0)\,\{\frac{r_0(1-r_0^a)}{(1-r_0)^2}\}]\,P_{1a-1}$..............(35)

Probability that one server is busy and the other server is on switchoverperiod (P_{3a-1})
If one of the servers observes that there are $a-1$ customers in the system while the other server is busy, it remains idle until the number of customers reaches a. This idle waiting time is referred to as the server's

ISSN 2394 - 7780

switchover period. Let P_{3a-1} denote the probability that one server is active and the other is in the switchover phase.

$P_{3a-1} = [A_1(r_1) + B(r_0)]\, P_{1a-1}$ ……………………………………………………………(36)

Probability that the server busy with accessible batch (Q_{3B})

When one server is occupied with a batch of n customers, where a < n < d − 1, the other server begins serving the remaining customers under the following conditions: if the queue has d to b customers, the server takes all of them for service; if the queue has more than b customers, it serves exactly b customers. This type of service arrangement is referred to as a Non-Accessible Batch Service. In this scenario, both servers are actively engaged—one with an accessible batch, and the other with a non-accessible batch.

$$Q_{3B} = (A_3 \frac{r_3(r_3^a - r_3^{d-1})}{(1-r_3)^2}] + AH_n(r_1) \frac{r_1(r_1^a - r_1^{d-1})}{(1-r_1)^2}\} + BH_n(r_0) \frac{r_0(r_0^a - r_0^{d-1})}{(1-r_0)^2} + kr_0^{n-a+2})\, P_{1a-1} \quad \ldots(37)$$

Numerical Analysis

The numerical results for the performance measures, based on selected values of the parameters a, b, θ, μ, and λ, are presented in Tables 1, 2, and 3. From Table 1, it is observed that the normalization condition $P_{0B} + P_{1B} + P_{2B} + P_{3a-1} + Q_{3B} \approx 1$ holds true across various combinations of a, b, and λ, confirming the consistency and validity of the probability distributions used in the model..

Table 1: The steady state results along with L_q for various values of $a,d, b, \theta = 0.2$ and μ= 1

λ		L_q	P_{0B}	P_{1B}	P_{2B}	P_{3a-1}	Q_{3B}
5	$a = 10$	4.9166	0.5117	0.4375	0.0001	0.000041	0.00001
10	$d = 13$	7. 2293	0.3323	0.6693	0.0015	0.004710	0.002110
15	$b = 25$	11.1211	0.2540	0.6800	0.0118	0.012060	0.011120
6	$a = 20$	8.9145	0.7674	0.1601	0.0008	0.000080	0.000071
12	$d = 25$	10.7853	0.4340	0.5245	0.0097	0.003421	0.002900
18	$b = 30$	15.2785	0.2100	0.6345	0.0678	0.004560	0.003950
10	$a = 30$	13.1418	0.7024	0.2921	0.0006	0.000041	0.000032
20	$d = 35$	18.6009	0.5106	0.4206	0.0021	0.001178	0.00232
30	$b = 50$	27.7541	0.3123	0.6076	0.0124	0.013140	0.01450

Table 2: L_q for various values of , a, d when b = 50, θ = 0.5 and μ= 1

λ	a=10 d = 15	a=20 d = 25	a=30 d = 35	a=40 d = 45
5	5.2399	9.0993	15.2845	19.2809
10	7.5620	11.6587	15.6054	19.9154
15	12.0918	12.7643	15.9769	20.1236
20	16.4365	15.8790	17.9896	21.4732
25	18.9994	19.4367	20.9076	22.8553

Cost Model

The cost analysis is done for the models analyzed in this chapter by considering different costs associated with the servers and customers waiting time. Let

C_0 = fixed cost per unit time for each server

W_0 = waiting cost per unit service by each server

C_1 = cost per unit service by each server

If M denotes the expected total cost per unit time for operatingthe system, then $M = 2C_0 + W_0 L_q + C_1\, \mu\, (2P_{2B} + P_{1B} + (a-1)P_{3a-1} + (d-1)\, Q_{3B})$. The expected total cost per unit time for the operating system M is compared with single vacation of M/M (a,b)/(2,1) for various values of a, b when θ= 0.1 and μ= 1

Table 3: Comparison of L_q and M for M/M (a,b) /(2,1) andM/M (a, d, b)/(2,1) model

		M/M(a,b)/(2,1) Single vacation			**M/M(a,d,b)/(2,1) singleand delayed vacation**	
		L_q	M		L_q	M
5		5.262475	75.069336		3.0001	71.8076
10	a=10	10.98572	92.09053	a=10	4.0111	77.2768
15	b=25	20.927956	118.796974	d=15	5.0043	101.6114

 ISSN 2394 - 7780

20		34.238483	161.142303	*b=25*	5.1201	118.12263
8	*a=25* *b=40*	12.347372	93.351715	*a=25* *d=32* *b=40*	10.7087	89.7553
16		16.032642	109.756927		11.0517	100.0012
24		26.168909	144.198929		11.9604	118.4390
32		46.064209	207.200989		12.0541	129.23140
10	*a=40* *b=50*	19.675701	114.010056	*a=40* *d=43* *b=50*	16.4839	100.3221
20		22.677698	127.722862		17.0023	115.7685
30		32.856942	162.452194		18.1805	125.44432
40		55.439793	233.982773		20.0135	145.64786

From the table 3 we infer that L_q in M/M(a,b)/(2,1) is more compared to M/M(a, d, b)/(2,1) queueing model.

Pictorial Representation

The graph illustrates a comparison of the expected queue length (Lq) between the proposed model and other existing vacation queueing models. It is evident that the average number of customers in the proposed M/M (a, d, b)/(2, 1) system is significantly lower than that in the traditional M/M(a,b)/(2,1) model.

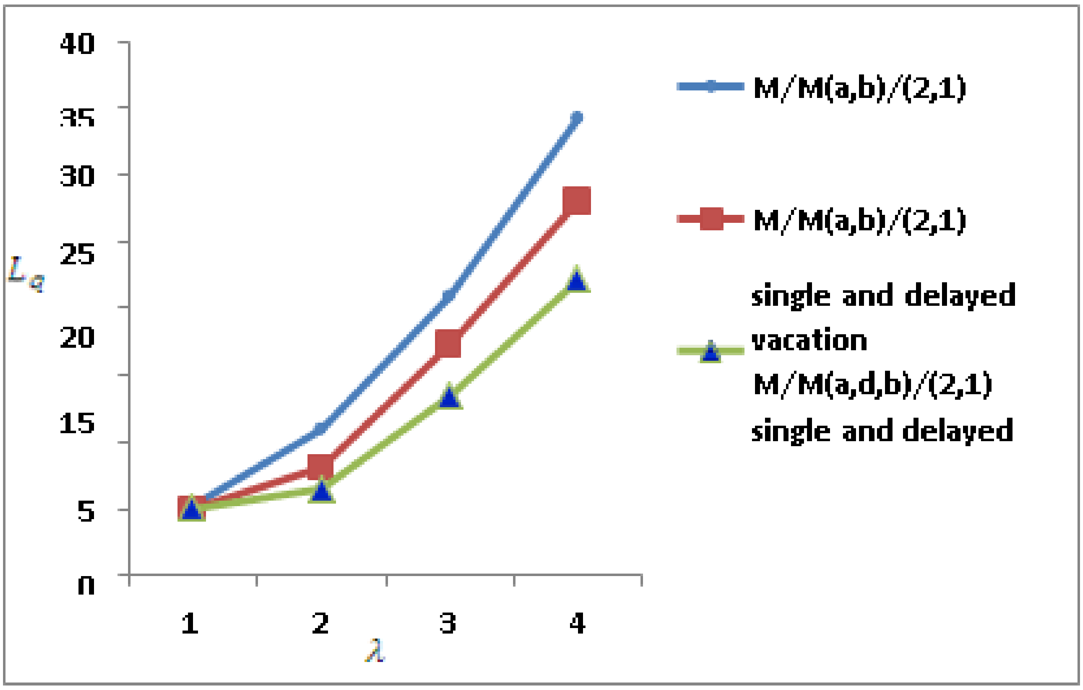

Figure 1: Comparison of M/M(a,b)/(2,1) and M/M(a,d,b)/(2,1)queueing models

CONCLUSION

In this study, an M/M(a,d,b)/(2,1) vacation queueing model is analyzed, where each server takes a single vacation based on batch sizes and the server's switchover period. Numerical results for key performance measures are examined across various parameter values. Table 3 presents a comparison of the expected queue length (Lq) for both delayed and single vacation scenarios, with graphical representation of the variations. It is observed that the proposed model results in a significantly lower Lq compared to existing models. This indicates a reduction in customer waiting time, primarily due to the inclusion of accessible units in the ongoing service batch. The accessible batch service not only improves efficiency but also offers more cost-effective and responsive service delivery within the system.

REFERENCES

1. Arumuganathan, R. and Malliga, J., Analysis of Bulk Queue with Repair of Service Station and Setup Time, Canadian Applied Mathematics Quarterly, Vol.13, No. 1, Spring, pp. 19-42, 2005.
2. Altman, E & Yechiali, U 2006, 'Analysis of customers' impatience in queues with server vacations', Queueing systems, vol. 52, no. 4, pp. 261-279.
3. Anitha, R 1997, 'Analysis of some bulk service queueing models with multiple vacation', Ph.D thesis, Bharathiar University.

ISSN 2394 - 7780

4. Arumuganathan, R & Ramaswami, K 2005, 'Analysis of a bulk queue with fast and slow service rates and multiple vacations', Asia-Pacific Journal of Operational Research, vol. 22, no. 02, pp. 239-260.
5. Ayyappan, G & Renganathan, N 1997, 'A preemptive priority queue with accessible batch service and heterogeneous arrivals', International journal of information and management sciences, vol. 8, no. 1, pp. 63-72.
6. Ayyappan, G, Sathiya, K & Subramanian, AMG 2013, 'Mx/G/1 Queue with Two Types of Service Subject to Random Breakdowns, Multiple Vacation and Restricted Admissibility', Applied Mathematical Sciences, vol. 7, no. 53, pp. 2599-2611.
7. Ayyappan, S & Sridevi, S 2012, 'Bulk service Markovian queue with service batch size dependent and accessible and non-accessible service batches and with vacation', Journal of Computations & Modelling, vol. 2, no. 1, pp. 123-135.
8. Baba, Y 1986, 'On the Mx/G/1 queue with vacation time', Operations Research Letters, vol. 5, no. 2, pp. 93-98.
9. Bailey, NTJ 1954, ‘On queueing process with bulk service’, Journal of Royal Statistical Society: Series B, vol.16, pp. 80-87.
10. Bagyam, JEA, Chandrika, KU & Rani, KP 2013, 'Bulk Arrival Two Phase Retrial Queueing System with Impatient Customers, Orbital Search, Active Breakdowns and Delayed Repair', International Journal of Computer Applications, vol. 73, no. 11.
11. Balasubramanian, M & Arumuganathan, R 2011, 'Steady state analysis of a bulk arrival general bulk service queueing system with modified M-vacation policy and variant arrival rate', International Journal of Operational Research, vol. 11, no. 4, pp. 383-407.
12. Borthakur, A & Choudhury, G 1997, 'On a batch arrival Poisson queue with generalized vacation', Sankhyā: The Indian Journal of Statistics, Series B, pp. 369-383.
13. Borthakur, A 1971, 'A Poisson queue with a general bulk service rule', J. Assam Sci. Soc, vol. 14, pp. 162-167.
14. Bruneel, H & Wuyts, I 1994, 'Analysis of discrete-time multiserver queueing models with constant service times', Operations Research Letters, vol. 15, no. 5, pp. 231-236.
15. Chakravarthy, SR 2007, 'A multi-server synchronous vacation model with thresholds and a probabilistic decision rule', European Journal of Operational Research, vol. 182, no. 1, pp. 305-320.
16. Chakravarthy, SR 2009, 'Analysis of a multi-server queue with Markovian arrivals and synchronous phase type vacations', Asia-Pacific Journal of Operational Research, vol. 26, no. 01, pp. 85-113.
17. Chen, A, Pollett, P, Li, J & Zhang, H 2010, 'Markovian bulk-arrival and bulk-service queues with state-dependent control', Queueing systems, vol. 64, no. 3, pp. 267-304.
18. Doshi, BT 1986, 'Queueing systems with vacations—a survey', Queueing systems, vol. 1, no. 1, pp. 29-66.
19. Easton, G & Chaudhry, M 1982, 'The queueing system Ek/M(a,b)/1 and its numerical analysis', Computers & Operations Research, vol. 9, no. 3, pp. 197-205.
20. Gupta, U & Laxmi, PV 2001, 'Analysis of the MAP/G(a,b)/1/N Queue', Queueing systems, vol. 38, no. 2, pp. 109-124.