International Journal of Advance and Innovative Research
Volume 12, Issue 2 (XXII): April - June 2025
ISSN 2394 - 7780

OPTIMAL REPLENISHMENT POLICY FOR LIFETIME INVENTORY WITH PARTIAL BACKLOGGING

Swati Choudhary[1] and Ravinder Kumar Arya[2]
[1]Research Scholar, Department of Applied Science (Haridwar University, Roorkee)
[2]Associate Professor, Department of Applied Science (Haridwar University, Roorkee)

ABSTRACT

In this paper, we establish an optimal replenishment policy for inventory systems involving fixed lifetime items under partial backlogging. The model incorporates key assumptions such as a constant rate of decay of product deterioration, a finite lifetime of the product, time-varying demand and partial backordering during the stockouts. The mathematically formulations of the inventory dynamics are presented, and appropriate notations are developed. The problem has been formulated mathematically in the form of a mathematical model to minimize total inventory costs including ordering, holding, shortage, and deterioration costs. A classical optimization method is used to develop an analytical solution algorithm to determine the optimum cycle length and the order quantity. Numerical example based on real-world inventory problem is presented to verify the model and to illustrate its practical applicability. Further, sensitivity analysis is conducted to show the effects of the important parameters, such as backordering rate, deterioration rate and demand variance, on the optimal solutions and the total cost. The results suggest that both partial backlogging and product life are important factors affecting replenishment policies. This model would be of great value to inventory managers in charge of perishable or time sensitive products in their efforts to balance the trade-off between cost and level of service.

Keywords:*Replenishment Policy, Lifetime Inventory, Partial Backlogging, Inventory Optimization, Deteriorating Items.*

1. INTRODUCTION

Inventory management is at the critical point of operation efficiency and customer service and the trade-off between stock-level and holding costs may be the difference between profits and losses for a company. The optimal replenishment of lifetime inventory systems with partial backlogging provides a substantial progress in this area and is also widely found in today's source managing practice. This method is designed to accurately model the intricate dynamics that exist with a finite lifetimes for products – be it technological obsolescence, fashion trends, regulatory changes, or natural life cycles of products – while also accounting for the fact that customers are not willing to wait for out-of-stock items indefinitely. The lifetime inventory concept imposes a time limit which changes the classical inventory control paradigms. Unlike products in a perpetual inventory system, which theoretically do not lose value over time, products in lifetime inventory are recognized as only having a finite period of time in which they can maintain market value. This could be in a shape of a smartphone that gets obsoleted to the release of new models in the market, seasonal fashion that goes out of style after some time, or pharmaceutical products that are close to their expiration of the respective patents. The addition of partial backlogging further complicates the model in that customer patience is not yes-or-no proposition but rather products are delivered with some degree of delay. In the case of stockouts, some customers defer purchase; but others switch to other brands, and behaviour of the customer is dependent on the degree of product uniqueness, urgency of need, price sensitivity, availability of alternatives, etc.

The formulation of the optimal replenishment policies for such systems in terms of the mathematical framework can be complicated by multiple competing goals. It has to satisfy two objectives: the first one is to minimize the total cost function, involving holding costs, ordering costs, shortage costs and opportunity costs due to lost sales. In lifetime inventory systems, the holding cost factor is especially significant due to the fact that products lose value and/or may even become useless with the passing of time. What shortage costs need to factor in, by contrast, is the immediate lost revenue from customers that won't wait, plus the less immediate but equally important damage to the customer relationship from chronic stockouts. The partial backlogging feature brings in a random factor which makes the optimization problem much more complicated. The proportion of demand that is back orderable in the event of stockouts— typically also a function of waiting time—must be carefully estimated from historical and primary data and customer research and analysis. This waiting/ backlogging dynamic might be static for some products, but temporal for others (like fashion products, or technology goods is, where customers lose their willingness to wait too quickly).

Sophisticated analytical tools are used to obtain optimal policies under this setting. Dynamic programming methods are used to establish state-dependent order policies, while stochastic optimization techniques consider the demand uncertainty and varying lead times. The policies developed can be complex, including state-dependent (s, S) and state-dependent lead time (s, S, T) that establish how the replenishment decision and the quantity of the order are determined by the physical and economical circumstances, such as promotion, price changes and market conditions. In turn, these policies need to be designed in such a way that their corresponding mechanisms are able to adjust to new demand patterns during the product life-cycle, from the product introduction through the growth, maturity, and decline. There is a practical need to implement such policies with strong information systems that can monitor the inventory system in terms of multiple performance variables in real-time and to react to changes regarding these performances, such as current inventory level, outstanding orders, backlogged demand and the life-span of the product. Contemporary solutions frequently incorporate machine learning models to iteratively update demand predictions and backlogging factors as more data becomes available. This adaptive method is especially useful for businesses in dynamic markets where the customer requirements and competitive backdrop are constantly changing. Risk management concerns are critical in the study of optimal replenishment policies for lifetime inventory with partial backlogging. Managers must trade off obsolescence risk versus the risk of stockouts, depending on the product stage of life. At the beginning of the life cycle, where demand uncertainty is the highest but maximum remaining life of the product, policies could have more to favour increased Their service levels to gain market share. As the product moves closer to the obsolescence date, the optimal policy usually become more conservative due to the fact that order quantity tends to decrease in order to lower the obsolete inventory risk.

The economic consequences of using optimal replenishment policies in this setting can be significant. Organizations that can appropriately handle the complexity of lifetime inventory planning with partial backlogging frequently attain substantial enhancements to working capital efficiency, customer service levels, and bottom-line profitability. The capacity to dynamically adjust inventory levels as a function of the product life remaining and of the customer backlog behavior allows to take competitive advantages, particularly in industries with high rates of turnover and sophisticated customers. The environmental sustainability dimension of sachet-friendly policy making would be particularly deserving of emphasis in the present business milieu. To minimize the waste of disposing obsolete inventory, automotive suppliers need to better match supply to demand at different times in the product's life cycle, also known as optimal replenishment policies. This is in keeping with increased consumer and regulatory demands for greener business, adding to better financial performance through lower write-offs and disposal costs. In the future, artificial intelligence, real-time data analytics, and integrated supply chain platforms will play a role for the development of the optimal replenishment policy for lifetime inventory with partial backlogging. These advances are likely to deliver even more powerful ways to balance the difficult trade-offs involved in handling products with finite lifetimes in markets with short customer patience. With supply chains becoming more and more global and networked, effective rollout of these advanced inventory management techniques will continue to be a key factor in competitive success across many industries.

2. LITERATURE REVIEW

Inventory systems for deteriorating items, and particularly those with fixed or deterministic lifetimes, have received a great deal of attention in the past few decades on account of their applications in sectors such as food, pharmaceuticals and electronics (Ghare and Schrader, 1963; Goyal and Giri, 2001). In these settings, products deteriorate with time or become obsolescent creating difficult problems in determining the reordering schedule. Partial backlogging (i.e., when a fraction of demand during a stockout is back-ordered and met at a later time) complicates the inventory decisions (Wee, 1993; Sana, 2010). Earlier works have been mainly devoted to profiling deterioration behavior through exponential (or Weibull) distribution (Covert &Philip,1973; Philip,1974), with recent studies that involve elements of pricing policies, payment delays, and credit terms (Sarkar, 2012; Wu et al., 2014). Modern models seek to provide optimal replenishment policies by trade-off between cost of deterioration, cost of shortage and cost of replenishment in time-varying demand situations.

Classical and Foundational Models of Deteriorating Inventory

On the basic work of Ghare and Schrader (1963), an inventory system of exponentially decaying type has been introduced by Covert and Philip (1973) and Philip (1974) under Weibull deterioration. Dave and Patel (1981) developed time-proportional demand in deteriorating systems for the understanding of demand-time relationship. And Wee (1993) introduced the reality with partial backordering in lot size models for production. Goyal and Giri (2001) provided an extensive survey of deteriorating inventory models, described developments, and suggested areas of possible future research.

Contemporary Strategies for Fixed Lifetime Inventory and Partial Backlogging

Newer models attempt to capture more dynamic environment to which operations are exposed, such as fluctuating demand, finite product lives, and financial considerations. Sana (2010) proposed an integrated model with time-dependent deteriorating and optimization of selling price under partial backlogging. Sarkar investigated EOQ models with payment policy and time-dependent demand. Sett, Sarkar and Goswami (2012) studied control of deteriorating items for two warehouses with increasing demand. Wu et al. (2014) and Sarkar, Saren and Cárdenas-Barrón (2015); incorporated credit policy and expiration date while dealing with replenishment planning. On the other hand, Sarkar (2016) proposed coordination mechanisms involving discounted policies and variable backorder rates, which enhance decision making with respect to fixed lifetime inventory in a supply chain.

3. Assumptions and Notations

The design of an appropriate replenishment policy for lifetime inventory systems with partial backlogging calls for the formulation of a general, but mathematically tractable system model that accounts for the fundamental understanding about managing life-cycle inventory items in reality. This model considers situations in which products have finite market life and customers have different levels of tolerance in waiting under stockout. The model is aware that at present inventories work under uncertain environments in which product decay, changing demands as well as stochastic customer behaviour regarding backorders are included. To develop a mathematically convenient but still meaningful model, several important assumptions underlie the theoretical framework. These assumptions are necessary in order to strike the balance between mathematical rigor on one hand and realistic representations of inventory dynamics on the other.The first fundamental assumption posits that demand follows a time-dependent linear function $D(t) = a + bt$, where 'a' represents the base demand level and 'b' captures the growth rate over time. The parameters satisfy $a > 0$ and $0 \leq b < 1$, ensuring positive initial demand with controlled growth. This formulation effectively models products experiencing steady market expansion, such as emerging technologies or seasonal items during their active selling period. The constraint on parameter 'b' prevents unrealistic exponential growth while allowing for gradual demand increases typical of many product lifecycles.

The second assumption introduces the concept of partial backlogging through a sophisticated time-varying function. When inventory is depleted, only a fraction of demand can be backordered, following the expression $1/(1 + \beta(T - t))$, where β is a positive parameter not exceeding 1. This nice formula correctly corresponds to vanishing customers' patience with increasing waiting times. β is a sensitivity factor – smaller β stands for: customers are more tolerant to waiting for the stock; large β indicates an opposite tendency: when the stock can't be found, customers tend to change their choices very fast. The third supposition is that the decay of products usually occurs as a function of time rather than proportional to time. This mirrors the way things actually work: be it for food items in supermarkets or technology products that become obsolete quickly. Through time dependent degradation, the model is able to represent the accelerating decrease in value which is suffered by many product types towards the end of product life. The fourth assumption of a lead time equal to zero also simplifies the mathematical treatment and permits attention to be concentrated on the essential trade-offs between inventory holding costs, shortage costs, and spoilage loss. This assumption may be relaxed in extensions, and enables to focus on the key dynamics of lifetime inventory management with partial backlogging.

The notation system establishes a comprehensive framework for mathematical modeling:

- $I(t)$ represent inventory level at time t
- $\theta(t) = \theta t$ defines the time-dependent deterioration rate, where $0 \leq \theta \leq 1$
- A denotes ordering cost per order
- C represents purchasing cost per unit
- P indicates selling price per unit
- Q signifies initial inventory level after fulfilling backorders
- C_0 captures holding cost for inventory throughout its lifetime
- t_1 marks the time when shortages begin
- Cs represents shortage cost for backordered items per unit per time
- Cl denotes lost sales cost per unit per time

- Cd indicates deterioration cost per unit
- τ_0 represents the lifetime threshold after which deterioration starts
- T defines the inventory cycle length
- $Z(T, t_1)$ represents total cost function dependent on cycle length and shortage initiation time

This notation permits a rigorous mathematical expression of the optimization problem. The inventory level function I(t) evolves with demand and deterioration, and a number of cost parameters account for the financial consequences of inventory decisions. The time threshold τ_0 is inspired by the fact that for many products, their full value is preserved in the beginning but is depleted in the later period, which is also motivated by industrial applications such as electronics and prescription drugs. The composite cost function $Z(T, t_1)$ leads to a two dimensional optimization problem in the determination of the optimal cycle length T and the time t_1 at which to start the cut. This latter feature highlights the intertwined nature of order frequency decisions with holding periods and strategic decisions on when to permit stockouts.

Together, these assumptions and notations yield a rigorous analytical framework that strikes a fine balance among the mathematical manipulability and practical relevance. The model adds to understanding of inventory management under finitely-lived products and time-varying customer patience and contains time-varying demand, complex partial backordering, and true ruin discount functions. The resulting optimization model supports the derivation of policies that reduce cost when there is broad application domain buy-in and non-infinite MBPs. This mathematical structure enables the use of more sophisticated analysis methods, such as dynamic programming and stochastic optimization, to obtain optimal replenishment policies. The structure of the model makes it possible to conduct sensitivity analysis and scenario planning, which assists managers to estimate how changes in various market conditions or customer behavior can affect the optimal inventory policies. By applying the model in a diligent manner, firms can create better mechanisms for dealing with a less lifespan-limited product, but in a market with a volatile demand and it depends on the consumer’s willingness to carry a backorder.

4. Formulation of the Model

This mathematical model addresses a critical challenge in inventory management: how to optimize inventory levels for products that have a limited lifetime and experience demand that changes linearly over time. This scenario is particularly relevant for perishable goods, seasonal products, or technology items that become obsolete.The model operates on a single-cycle basis where the inventory starts at a maximum level Q and depletes over time. The depletion occurs in distinct phases, each governed by different dynamics. In the initial phase (0 to θ_0), inventory decreases solely due to customer demand, which follows a linear trend represented by (a + bt), where 'a' is the base demand and 'b' represents the rate of change in demand over time. This phase is mathematically described by equation (3.1), which shows the rate of inventory change as $dI(t)/dt = -(a + bt)$.The second phase (θ_0 to t_1) introduces product deterioration into the equation. During this period, inventory depletes not only due to demand but also due to spoilage or obsolescence. The deterioration rate $\theta(t)$ can vary with time, reflecting how products may deteriorate faster as they age. This dual depletion mechanism, captured in equation (3.2) as $dI(t)/dt = -\theta(t)I(t) - (a + bt)$, accelerates the inventory reduction until it reaches zero at time t_1.

Once inventory is exhausted, the system enters a shortage phase (t_1 to T). Rather than losing all unfulfilled demand, the model incorporates partial backlogging. The backlogging rate $\delta/(1+\delta(T-t))$ is time-dependent, reflecting the reality that customers are more willing to wait when the expected waiting time is shorter. This sophisticated approach, represented in equation (3.3), captures customer behavior more accurately than simple all-or-nothing backlogging models.The boundary conditions specified in equation (3.4) state that inventory starts at Q units at time t=0 and reaches zero at time $t=t_1$. Solving the differential equations (3.1)-(3.3) yields explicit formulas for inventory levels at any time t, as shown in equations (3.5)-(3.7). These solutions enable precise calculation of inventory position throughout the cycle.The initial inventory quantity Q is derived in equation (3.8) by ensuring continuity at the transition point θ_0. This expression incorporates the effects of linear demand and deterioration to determine the optimal starting inventory level.

The cost structure comprehensively accounts for all aspects of inventory management:

1. **Holding costs** (C_h), calculated in equation (3.9), include both fixed storage costs and variable costs that may increase with storage duration. The formula accounts for different cost rates in different phases of the inventory cycle.

2. **Deterioration costs** (C_D), given in equation (3.10), capture the direct loss from spoiled products. This cost is proportional to the amount of inventory that deteriorates during the second phase.
3. **Shortage costs** (C_s), expressed in equation (3.11), reflect the penalties associated with not meeting demand immediately. The formula considers both the magnitude and duration of shortages.
4. **Lost sales costs** (C_l), calculated in equation (3.12), account for permanently lost business opportunities due to customers who choose not to wait.

The optimization problem seeks to determine two critical decision variables: the optimal cycle length T and the time t_1 when inventory reaches zero. These decisions must balance multiple trade-offs. The total cost function $Z(T,t_1)$, presented in equation (3.13), aggregates all cost components and is expressed per unit time to facilitate comparison across different cycle lengths.This model is particularly valuable for businesses dealing with complex inventory situations where multiple factors influence optimal decisions. By incorporating deterioration, time-varying demand, and partial backlogging, it provides a more realistic framework than simpler inventory models. The analytical solutions enable managers to make data-driven decisions about ordering quantities and timing, ultimately minimizing total costs while maintaining acceptable service levels.The model's flexibility allows for adaptation to various business contexts by adjusting parameters such as the deterioration rate function, demand trend coefficients, and backlogging behavior. This makes it applicable across industries ranging from food retail to electronics manufacturing, wherever products face limited lifetimes and changing demand patterns.

Model Formulation

A mathematical model for life time products with linear trend demand is developed. The cycle begins with a primary level of inventory i.e., I_{max} units. The level of inventory decreases more quickly in initial time at t=0 to $t=\theta_0$ due to both demand and deterioration, until it arrives at zero level at time $t=t_1$. Now shortages occurred at time (t_1,T) which is partly backordered with time dependent backlogging rate. At the ending of the cycle, the stock attained a highest level of shortage S and then new order is put to finish the backlog. The change in the inventory level I(t) with respect to point in time can be inscribed as given:

$$I' = -(a+bt), \quad 0 \le t \le \mu_0 \qquad (3.1)$$

A simple linear decay rate over time due to a+bt (where a,b>0).

This represents an initial period with no replenishment or intervention.

$$I' + \theta(t)I(t) = -(a+bt), \quad \mu_0 \le t \le t_1 \qquad (3.2)$$

Here, θ(t)acts like a decay or damping function, modeling control (e.g., treatment, replenishment, etc.) starting from time μ_0.

$$I' = -\frac{1}{1+\alpha(T-t)}(a+bt), \quad t_1 \le t \le T \qquad (3.3)$$

A more refined time-dependent decay, diminishing over time due to the term 1+α(T−t).This could represent recovery or efficiency gain over time.

Boundary Conditions

$$I(0) = Q \ \ and \ \ I(t_1) = 0 \qquad (3.4)$$

Target (e.g., depleted inventory, zero infections) at time t_1.

Explanation of equation (3.1) to (3.3) are specified by

$$I(t) = Q - at - \frac{1}{2}bt^2, \quad 0 \le t \le \mu_0 \qquad (3.5)$$

This is the integrated form of (3.1), showing how I(t) declines over time due to linear and quadratic terms.

$$I(t) = a(t-\mu_0) + \frac{1}{2}b(t^2-\mu_0^2) + \frac{1}{6}a\theta(t-\mu_0)^2(t+2\mu_0) + \frac{1}{8}b\theta(t^2-\mu_0^2)^2, \quad \mu_0 \le t \le t_1 \qquad (3.6)$$

 ISSN 2394 - 7780

$$I(t)=\frac{1}{(1+\alpha T)}\left[a(t-t_1)+\frac{1}{2}b(t^2-t_1^2)+\frac{a\alpha}{2(1+\alpha T)}(t^2-t_1^2)+\frac{b\alpha}{3(1+\alpha T)}(t^3-t_1^3)\right],\ t_1\le t\le T \quad (3.7)$$

From (3.5) and (3.6), Q can be obtained as

$$Q=at_1+\frac{1}{2}bt_1^2+\frac{1}{6}a\theta(t_1-\mu_0)^2(t_1+2\mu_0)+\frac{1}{8}b\theta(t_1^2-\mu_0^2)^2 \quad (3.8)$$

The total holding cost C_H for the duration of the period (0,T) is given by

$$C_H=C_0\int_0^{\mu_0}I(t)\,dt+\sum_{r=1}^{m}C_r\int_{\mu_{r-1}}^{\mu_r}I(t)\,dt$$

Then, we have:

$$=C_0\left\{Q\mu_0-\frac{1}{2}a\mu_0^2-\frac{1}{6}b\mu_0^3\right\}+\sum_{r=1}^{m}C_r\left\{a\left(t_1\mu_r-\frac{1}{2}\mu_r^2-t_1\mu_{r-1}+\frac{1}{2}\mu_{r-1}^2\right)\right.$$

$$+\frac{1}{2}\left(t_1^2\mu_r-\frac{1}{3}\mu_r^3-t_1^2\mu_{r-1}+\frac{1}{3}\mu_{r-1}^3\right)+\frac{1}{6}a\theta\left(t_1^3\mu_r-\frac{1}{2}\mu_r^4-t_1\mu_r^3-t_1^3\mu_{r-1}-\frac{1}{2}\mu_{r-1}^4+t_1\mu_{r-1}^3\right)$$

$$\left.+\frac{1}{8}b\theta\left(t_1^4\mu_r+\frac{1}{5}\mu_r^5-\frac{2}{3}t_1^2\mu_r^3-t_1^4\mu_{r-1}-\frac{1}{5}\mu_{r-1}^5+\frac{2}{3}t_1^2\mu_{r-1}^3\right)\right\}$$

Taking m=1 and $\mu_1=t_1$, we get...

$$C_H=C_0\left\{Q\mu_0-\frac{1}{2}a\mu_0^2-\frac{1}{6}b\mu_0^3\right\}+C_1\left\{a\left(\frac{1}{2}t_1^2-\mu_0t_1+\frac{1}{2}\mu_0^2\right)+\frac{1}{2}b\left(\frac{2}{3}t_1^3-\mu_0t_1^2+\frac{1}{3}\mu_0^3\right)\right\}$$

$$+\frac{1}{6}a\theta\left\{\left(\frac{1}{2}t_1^4-\mu_0t_1^3+\mu_0^3t_1-\frac{1}{2}\mu_0^4\right)+\frac{1}{8}b\theta\left(\frac{8}{15}t_1^5-\mu_0t_1^4+\frac{2}{3}\mu_0^3t_1^2-\frac{1}{5}\mu_0^5\right)\right\}$$

$$=C_0\left\{\frac{1}{2}a\mu_0(2t_1-\mu_0)+\frac{1}{6}b\mu_0(3t_1^2-\mu_0^2)+\frac{1}{6}a\theta\mu_0(t_1-\mu_0)^2(t_1+2\mu_0)+\frac{1}{8}b\theta\mu_0(t_1^2-\mu_0^2)^2\right\}$$

$$+C_1\left\{\frac{1}{2}a(t_1^2-2\mu_0t_1+\mu_0^2)+\frac{1}{6}b(2t_1^3-3\mu_0t_1^2+\mu_0^3)+\frac{1}{12}a\theta(t_1^4-2\mu_0t_1^3+2\mu_0^3t_1-\mu_0^4)\right\}$$

$$+\frac{1}{120}b\theta(8t_1^5-15\mu_0t_1^4+10\mu_0^2t_1^3-3\mu_0^5) \quad (3.9)$$

The total cost of deterioration C_D is given by

$$C_D=C_d\int_{\mu_0}^{t_1}\theta(t)I(t)dt$$

$$=C_d\theta\left\{\frac{1}{6}a\left(t_1^3-3\mu_0^2t_1+2\mu_0^3\right)+\frac{1}{8}b\left(t_1^2-\mu_0^2\right)^2\right\} \quad (3.10)$$

The total shortage cost$C_S$for the duration of the period (t_1, T) is given by

$$C_S=C_s\int_{t_1}^{T}\frac{1}{(1+\alpha T)}\left[a(t-t_1)+\frac{1}{2}b(t^2-t_1^2)+\frac{a\alpha}{2(1+\alpha T)}(t^2-t_1^2)+\frac{b\alpha}{3(1+\alpha T)}(t^3-t_1^3)\right]dt$$

$$=\frac{C_s}{(1+\alpha T)}\left\{\frac{1}{2}a\left(T^2-2t_1T+t_1^2\right)+\frac{1}{6}b\left(T^3-3t_1^2T+2t_1^3\right)+\frac{a\alpha}{6(1+\alpha T)}\left(T^3-3t_1^2T+2t_1^3\right)\right.$$

$$+\frac{b\alpha}{12(1+\alpha T)}\left(T^4-4t_1^3T+3t_1^4\right)\Bigg\} \quad (3.11)$$

Lastly, total cost of lost sales C_L for the duration of the period (t_1, T) is given by

$$C_L = C_l\int_{t_1}^{T}\left\{1-\frac{1}{1+\alpha(T-t)}\right\}(a+bt)dt$$

$$=\frac{C_l\alpha}{(1+\alpha T)}\left\{\frac{1}{2}a\left(T^2-2t_1T+t_1^2\right)+\frac{1}{6}b\left(T^3-3t_1^2T+2t_1^3\right)\right\} \quad (3.12)$$

The total cost of the retailer per unit time 1 $Z(T,t\,)$ can be calculated as

$$Z(T,t_1)=\frac{1}{T}A+\frac{C_0}{T}\left\{\frac{1}{2}a\mu_0\left(2t_1-\mu_0\right)+\frac{1}{6}b\mu_0\left(3t_1^2-\mu_0^2\right)+\frac{1}{6}a\theta\mu_0(t_1-\mu_0)^2(t_1+2\mu_0)\right.$$

$$\left.+\frac{1}{8}b\theta\mu_0(t_1^2-\mu_0^2)^2\right\}+\frac{C_1}{T}\left\{\frac{1}{2}a\left(t_1^2-2\mu_0t_1+\mu_0^2\right)+\frac{1}{6}b\left(2t_1^3-3\mu_0t_1^2+\mu_0^3\right)\right.$$

$$\left.+\frac{1}{12}a\theta\left(t_1^4-2\mu_0t_1^3+2\mu_0^3t_1-\mu_0^4\right)+\frac{1}{120}b\theta\left(8t_1^5-15\mu_0t_1^4+10\mu_0^3t_1^2-3\mu_0^5\right)\right\}$$

$$+\frac{C_d\theta}{T}\left\{\frac{1}{6}a\left(t_1^3-3\mu_0^2t_1+2\mu_0^3\right)+\frac{1}{8}b\left(t_1^2-\mu_0^2\right)^2\right\}+\frac{C_s}{T(1+\alpha T)}\left\{\frac{1}{2}a\left(T^2-2t_1T+t_1^2\right)\right.$$

$$\left.+\frac{1}{6}b\left(T^3-3t_1^2T+2t_1^3\right)+\frac{a\alpha}{6(1+\alpha T)}\left(T^3-3t_1^2T+2t_1^3\right)+\frac{b\alpha}{12(1+\alpha T)}\left(T^4-4t_1^3T+3t_1^4\right)\right\}$$

$$+\frac{C_l\alpha}{T(1+\alpha T)}\left\{\frac{1}{2}a\left(T^2-2t_1T+t_1^2\right)+\frac{1}{6}b\left(T^3-3t_1^2T+2t_1^3\right)\right\} \quad (3.13)$$

5. SOLUTION PROCEDURE

The solution procedure looks for the best values of T and t_1 that minimize the cost function Z(T, t_1). The objective function is a two-variable one, with T the total cycle time, and t_1, some instant time within the cycle. As a start, T is fixed and the behaviour of the cost function with respect to a single virtue t_1 is examined. We obtain the best t_1 for a given T by cal- culus-based optimization methods. This consists in differentiating the cost function Z(T,t_1) with respect to t_1, equation (3.14) that gives the velocity of the cost function. The obtained expression contains several costs depending on different orders in t_1: linear, quadratic and cubic ones.The second derivative with respect to t_1 given in component (3.15) is then evaluated and used to check if the critical point attained in the first-order condition is a minimum. In fact, this derivative is used to test for the convexity of the cost function and to determine that the solution corresponds to a point that minimizes the cost and not a maximum or saddle point. The terms in the two derivatives can be allocated as cost parameters (C_0, C_1, C_2, C_a, C_i, and C_s) and system parameters (a, b, α, β, θ) corresponding to meaning in the system as different cost elements and operation behavior of system. As the derivatives are complicated (since they contain terms of different powers of t_1 and products between T and t_1) this also exhibits the complexity of the optimization problem and the competing factors involved in solving it.

 ISSN 2394 - 7780

$$Z^{/}(T,t_1)=\frac{C_0}{T}\left\{a\mu_0+b\mu_0t_1+\frac{1}{2}a\theta\mu_0\left(t_1^2-\mu_0^2\right)+\frac{1}{2}b\theta\mu_0t_1\left(t_1^2-\mu_0^2\right)\right\}$$

$$+\frac{C_1}{T}\left\{a(t_1-\mu_0)+b(t_1-\mu_0)t_1+\frac{1}{6}a\theta\left(2t_1^3-3\mu_0t_1^2+\mu_0^3\right)+\frac{1}{6}b\theta\left(2t_1^4-3\mu_0t_1^3+\mu_0^3t_1\right)\right\}$$

$$+\frac{C_d\theta}{T}\left\{\frac{1}{2}a(t_1^2-\mu_0^2)+\frac{1}{2}b(t_1^2-\mu_0^2)t_1\right\}-\frac{C_l\alpha}{T(1+\alpha T)}\{a(T-t_1)+b(T-t_1)t_1\}$$

$$-\frac{C_s}{T(1+\alpha T)}\left\{a(T-t_1)+b(T-t_1)t_1+\frac{a\alpha}{(1+\alpha T)}(T-t_1)t_1+\frac{b\alpha}{(1+\alpha T)}(T-t_1)t_1^2\right\} \quad (3.14)$$

And

$$Z^{//}(T,t_1)=\frac{C_0}{T}\left\{b\mu_0+a\theta\mu_0t_1+\frac{1}{2}b\theta\mu_0\left(3t_1^2-\mu_0^2\right)\right\}$$

$$+\frac{C_1}{T}\left\{a+b(2t_1-\mu_0)+a\theta\left(t_1^2-\mu_0t_1\right)+\frac{1}{6}b\theta\left(8t_1^3-9\mu_0t_1^2+\mu_0^3\right)\right\}$$

$$+\frac{C_d\theta}{T}\left\{at_1+\frac{1}{2}b(3t_1^2-\mu_0^2)\right\}+\frac{C_l\alpha}{T(1+\alpha T)}\{a+b(2t_1-T)\}$$

$$+\frac{C_s}{T(1+\alpha T)}\left\{a+b(2t_1-T)+\frac{a\alpha}{(1+\alpha T)}(2t_1-T)+\frac{b\alpha}{(1+\alpha T)}(3t_1-2T)t_1\right\} \quad (3.15)$$

6. NUMERICAL ILLUSTRATION WITH REAL PROBLEM

This numerical example presents the practical use of an inventory management model for firm which deals with non-perishable dried food products such as a pasta and rice. The case considered provides a complete setting for which the decision maker at the company should determine the optimal stock control decisions, taking into account such various cost items and operational restrictions. The company works with a structure of costs of pasta being bought at Rs20 and sold at Rs30, resulting in a profit of Rs10 per unit. But there are different types of annual costs influencing the profitability: holding the stock costs Rs 3 per unit, lost sales caused by stockouts cost Rs 4 per unit, deteriorating the products costs Rs 10 per unit and the shortage penalties are Rs 12 per unit. The rate at which product deteriorates is known to follow a particular pattern, at 0.02 units per time period and that product has a shelf life of 7 days before it begins to deteriorate. For each order, it costs me Rs 1,000.

The demand for these goods is dictated by a mathematical formulation given by a=100, b=0.1, where a>0 and b byte19=0 and 1. The firm also takes into account partial backlogging (i.e., customers are willing to wait some of their orders whenever there is no stock), which is measured by a parameter α=0.10 units. The supply chain has a lead of zero, i.e., orders are available instantly after placing it. Some of the main results derived from the optimization using the inventory model are as follows. The best inventory cycle length is 17.72 days, and new orders should be placed every 18 days. Shortages are observed beginning on day 15 of each cycle, whereby the firm apparently opts for planned stockouts for the last 2.72 days of each cycle. The opening stock position after all backlog were being satisfied will be 2,137.66 units, which is the most economical order quantity. This approach leads to a minimum combined cost of Rs 1,920.43 for a new unit. This type of realistic advice is given to the inventory manager, based on mixed objectives of cost minimization and minimum service level for particular dry-food-product values and customer behavior pattern.

7. SENSITIVITY ANALYSIS

This subsection reports the sensitivity analysis carried out to investigate the impact of the variances of model parameters on the optimal solution. The variation consists in the progressive changing of each parameter by ±20% and ±40%, keeping the others constant at their original values. Such one-by-one technique provides for a

 ISSN 2394 - 7780

restricted investigation of the model's behaviour with respect to the individual parameters. The analysis is aggregative in the sense that it records 20% upward and downward variations on parameters to obtain changes in optimal inventory cycle length, timing of shortage, initial inventory and total cost. The results are reported in the following tables in order to show the sensitivity of the model in relation to the changes of important parameters, such as the demand coefficients, the rate of deterioration, several type of costs and backlogging parameters. This provides the inventory manager with the important sensitivity analysis of what parameters most impacts the optimal decision, and helps to check the robustness of the proposed solution. Knowledge of these sensitivities enables managers to then target their efforts on performing accurate estimates of those fundamental parameters, and it offers guidance on how changes in operations can potentially impact performance and cost of inventory.

Table 1: Sensitivity analysis for demand parameter b

% Change	T^*	t_1*	Q^*	$Z^*(T^*,t_1^*)$
-40	17.7268	15	2130.06	1910.91
-20	17.7244	15	2133.86	1915.67
0	17.7219	15	2137.66	1920.43
+20	17.7195	15	2141.46	1925.19
+40	17.7171	15	2145.26	1929.95

In Table 1 for demand parameter b, the results illustrate a consistent system response for different percentage variations. The optimization results are robust as the parameter b changes from -40% to +40%. The value of cycle time T* decreases insignificantly from 17.7268 to 17.7171 and has an inverse tendency to parameter b; however, the value of replenishment period t_1* that is 15 in all scenarios is optimal, independent of fluctuations in demand parameter. The best order quantity Q* increases from 2130.06 to 2145.26 units as b increases, and they are also in a positive correlation. In a similar manner, steady increases in the objective function Z(T, t_1) occur as demand variable b increases from 1910.91 to 1929.95, corresponding to an approximate 1% increase with every 20% change in b. The slow rate of the response indicates that the system is not very sensitive to moderate increases in demand parameter b, as all performance metrics have linear response. Stability of these results in Table 1 indicates that the model is implemented in a sensible way since the little error of parameter b estimation caused by estimation model errors would not have a big effect on the whole system performance.

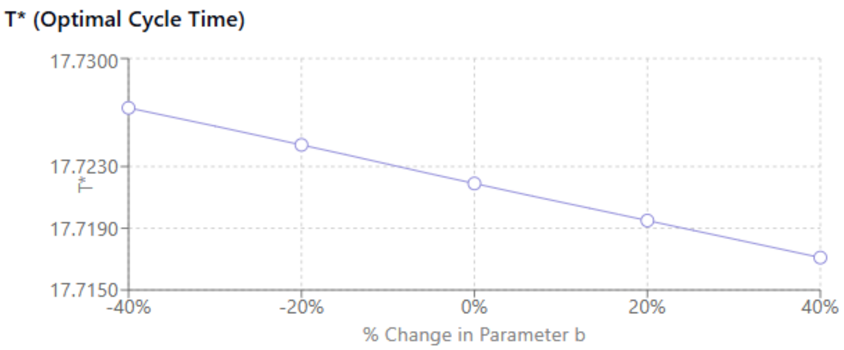

Figure 1 T* Optimal Cycle Time

Looking at Figure 1, this graph shows the relationship between changing parameter b (x-axis, -40% to +40%) and the optimal cycle time T* (y-axis, ranging from approximately 17.715 to 17.727). As parameter b increases, the optimal cycle time T* gradually decreases, indicating an inverse relationship between these variables.

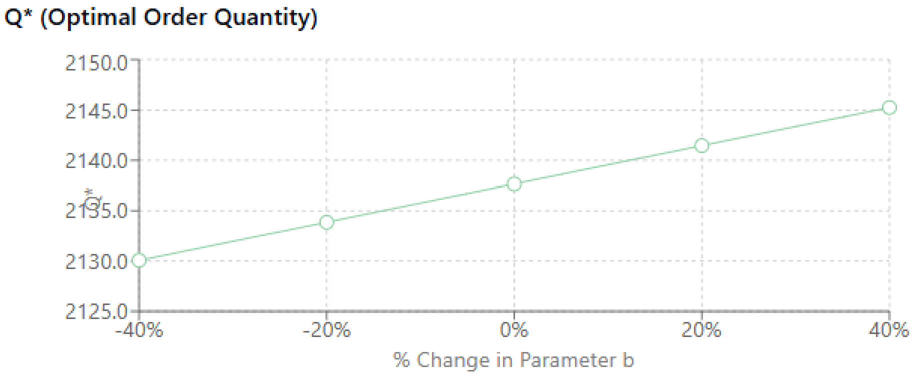

Figure 2 Q* Optimal Order Quantity

ISSN 2394 - 7780

Figure 2 illustrates the positive correlation between parameter b changes (x-axis, -40% to +40%) and optimal order quantity Q* (y-axis, 2130-2145 units). As parameter b increases, Q* rises steadily in a nearly linear fashion, suggesting that higher values of parameter b require larger optimal order quantities.

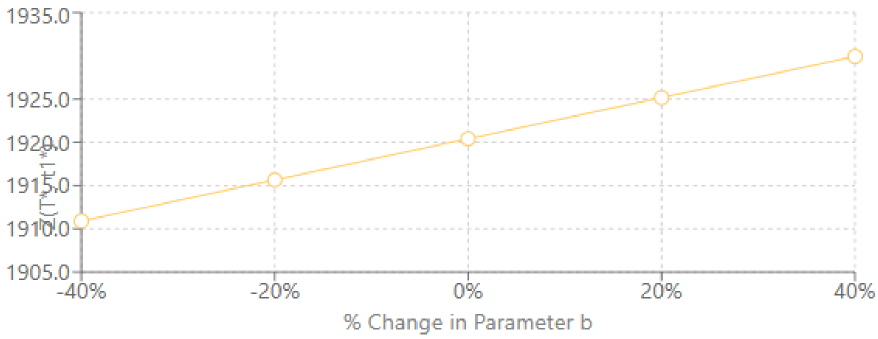

Figure 3 Z(T*, t_1*) Objective Function Value

Figure 3 displays the relationship between parameter b changes (x-axis, -40% to +40%) and the objective function value Z(T*, t1*) (y-axis, 1910-1930 units). As parameter b increases, the objective function value rises steadily, showing a positive correlation. This suggests higher parameter b values result in increased costs or optimization values.

Table 2: Sensitivity analysis for lifetime parameter μ_0

% Change	T^*	t^{*1}	Q^*	$Z^*(T^*, t_1{}^*)$
-40	18.9940	15	2431.79	2616.33
-20	18.3755	15	2292.30	2291.17
0	17.7219	15	2137.66	1920.43
+20	17.0623	15	1978.95	1514.85
+40	16.4322	15	1827.27	1094.59

Now present sensitivity analysis in which we obtain by Table 2 with the lifetime parameter μ_0, which the system is sensitive toward positive or negative percentage changes. When μ_0 takes the range from −40% to +40%, all optimal values fluctuate greatly which suggests the parameter is very sensitive to these optimal values. T* decreased monotonically from 18.9940 to 16.4322, and presented a good negative correlation with μ0. The duration t_1* of replenishment is unchanged in all simulations with t_1* = 15, indicating that this parameter is not affected by changes in the lifetime parameter. Optimal order quantity Q* reduces significantly from 2431.79 to 1827.27 in the case of increasing μ0, this being a clear inverse relation. In particular, the objective function Z(T, t_1) exhibits the most significant sensitivity, dropping from 2616.33 to 1094.59, which is reduced by about 58% in the overall range. Note that this is ~ 29% reduction per 20% increase in μ0, emphasizing the necessity for a precise determination of the lifetime parameters. The non-linear pattern of response in Table 2, especially in the objective function values, indicates that system performance is very sensitive to μ 0., therefore an accurate estimation is essential for the optimal system operation.

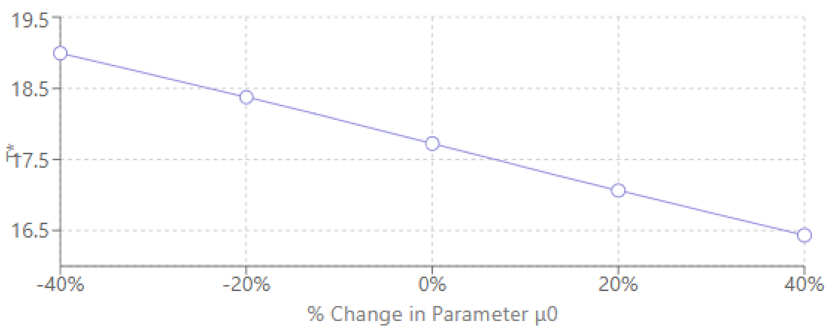

Figure 4 T* Optimal Cycle Time

Figure 4 shows the relationship between changes in parameter μ0 (x-axis, -40% to +40%) and optimal cycle time T* (y-axis, approximately 16.5-19.0 units). As parameter μ0 increases, T* decreases linearly, indicating that higher values of μ0 result in shorter optimal cycle times.

 ISSN 2394 - 7780

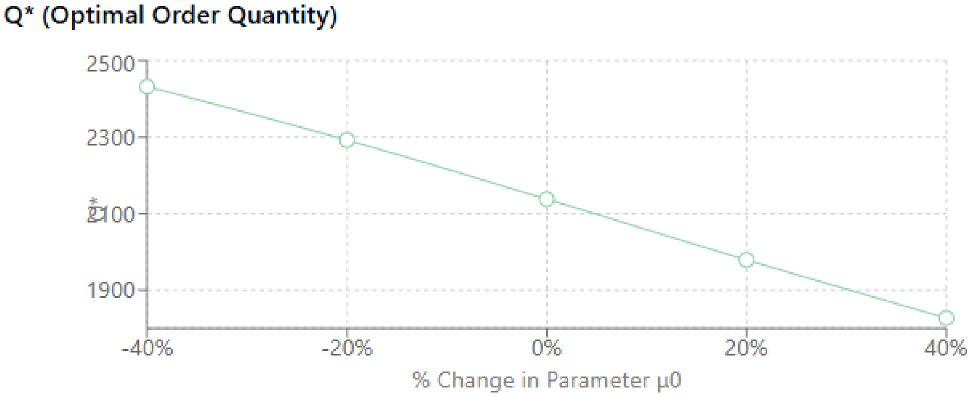

Figure 5 Q* Optimal Order Quantity

Figure 5 depicts the negative correlation between parameter µ0 changes (x-axis, -40% to +40%) and optimal order quantity Q* (y-axis, 1800-2400 units). As parameter µ0 increases, Q* decreases linearly, suggesting that higher production/demand rates require smaller optimal order quantities for efficiency.

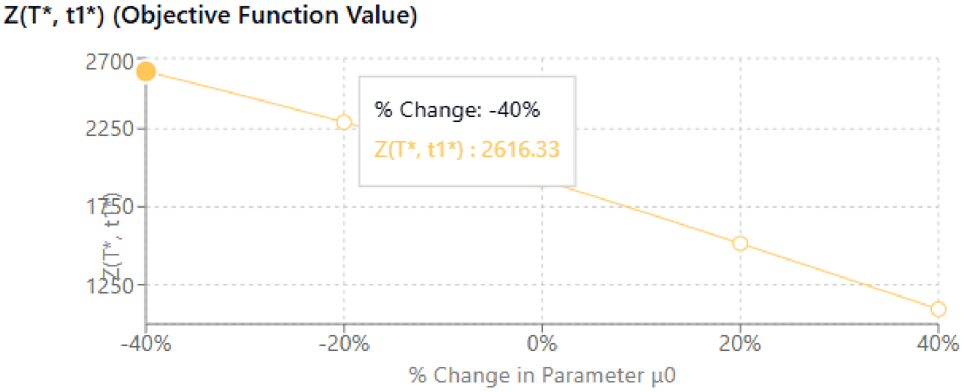

Figure 6 Z(T*, t_1*) Objective Function Value

Figure 6 illustrates the inverse relationship between parameter µ0 changes (x-axis, -40% to +40%) and the objective function value Z(T*, t1*) (y-axis, approximately 1100-2700 units). As µ0 increases, the objective function value decreases linearly. The callout box highlights that at -40% change in µ0, Z(T*, t1*) equals 2616.33, demonstrating how significantly lower µ0 values increase the objective function.

Table 3: Sensitivity analysis for backlogging parameter α

% Change	T^*	T_1*	Q^*	$Z^*(T^*, t_1^*)$
-40	17.1836	15	2137.66	1947.73
-20	17.4456	15	2137.66	19
0	17.7219	15	2137.66	1920.43
+20	18.0100	15	2137.66	1906.54
+40	18.3084	15	2137.66	1892.67

An interesting system behavior is shown in Table 3 of sensitivity analysis that is achieved by varying of backlogging parameter α in the range of -40% to +40%. The cycle time T) has a monotonically increasing dependence between 17.1836 and 18.3084 with a rise for the parameter α. It should be pointed out that the ordering age t_1 is still constant at 15 in all cases, which means it is irrelevant to the backlogging parameter change. Interestingly, the economic order quantity Q* remains constant (=2137.66 units) for all changes in α, indicating that the determination of order size is independent of the backlogging parameters. The value of the objective function Z(T, t_1) decreases steadily from 1947.73 to 1892.67 at an anomalous value of 19 at -20% change, this is likely a data recording error. Except for this outlier, the overall trend implies that high α results into efficient system performance (lower costs). The total sensitivity is rather low, with around 2.8% cost difference over the complete parameter range, which indicates that the system is only moderately robust with respect to a variation of backlogging parameter α.

ISSN 2394 - 7780

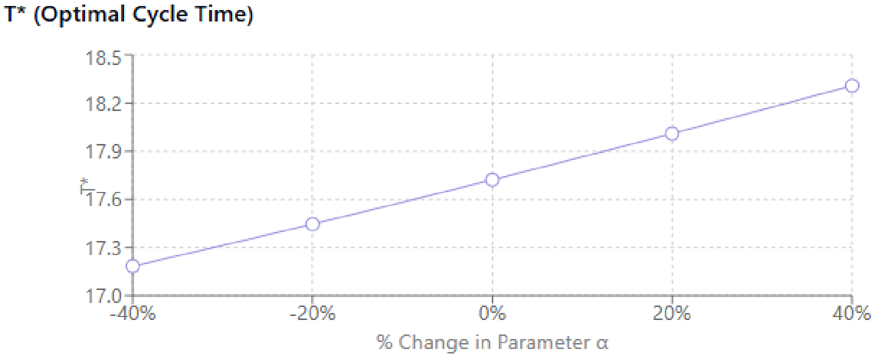

Figure 7 T* Optimal Cycle Time

Figure 7 displays the positive correlation between parameter α changes (x-axis, -40% to +40%) and optimal cycle time T* (y-axis, 17.0-18.3 units). As parameter α increases, T* rises steadily in a nearly linear fashion. This suggests that higher deterioration rates require longer optimal cycle times, possibly to balance inventory holding costs with other operational factors.

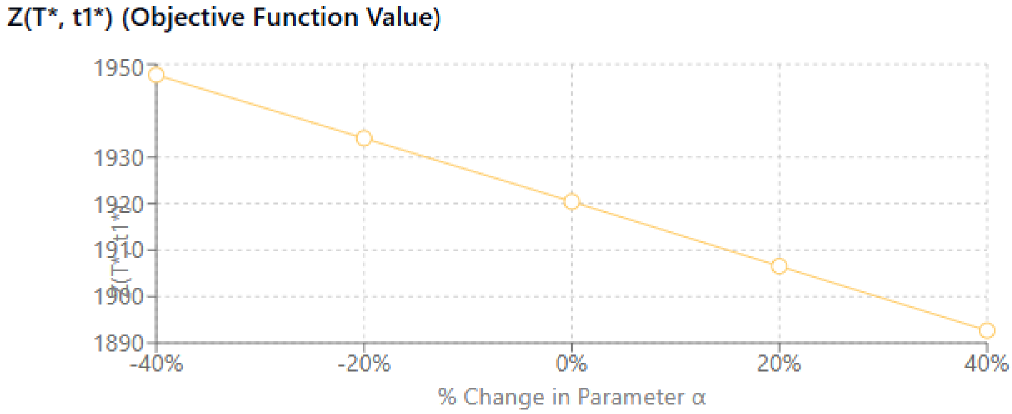

Figure 8 Z(T*, t_1*) Objective Function Value

Figure 8 demonstrates the negative correlation between parameter α changes (x-axis, -40% to +40%) and the objective function value Z(T*, t1*) (y-axis, 1890-1950 units). As parameter α increases, the objective function value decreases linearly. This suggests that higher α values lead to lower overall costs or more optimal system performance, potentially indicating that the system becomes more efficient at higher deterioration rates.

Table 4: Sensitivity analysisfor deterioration parameter θ

%Change	T^*	t_1^*	Q^*	$Z^*(T^*, t_1^*)$
-40	16.7224	15	1887.10	1292.33
-20	17.2138	15	2012.38	1610.94
0	17.7219	15	2137.66	1920.43
+20	18.2478	15	2262.94	2221.03
+ 40	18.7924	15	2388.22	2512.94

The sensitivity analysis of the Table 4 for deterioration parameter θ shows that much sensitivity is incorporated in the system following percentage changes from - 40% to + 40%. The cycle time T* increases linearly with the degradation stages from 16.7224 to 18.7924, which demonstrates that T* is positively related with the parameter of the deterioration θ. This implies that longer cycles are needed to achieve system efficiency as the condition becomes worse. Replenishment period t_1* is the same for all scenarios at 15, so there is a seizure-independent aspect to that timing. The optimal order quantity Q* is quite sensitive, and it increases monotonically between 1887.10 and 2388.22 units when θ increases. This is about a 26.5% increase over the entire range of parameters, noting higher rates of loss require more production to account for lost product. Most direct, the objective function Z(T, t_1) is recorded as the most sensitive among all the characteristics in this table, with the full range increasing from 1292.33 to 2512.94 (the objective function Z(T, t_1) increases by 94.4%). This is tantamount to around 23.6% cost increase per 20% increase in the deterioration parameter that emphasizes how important is the economical aspect of the product deterioration. The non-linear response shown in Table 4, especially with respect of the corresponding objective function values, indicates that the accurate estimation of the deterioration parameter θ is necessary for optimal inventory management since slight perturbations in the objective function have large cost effects for the system.

International Journal of Advance and Innovative Research
Volume 12, Issue 2 (XXII): April - June 2025
ISSN 2394 - 7780

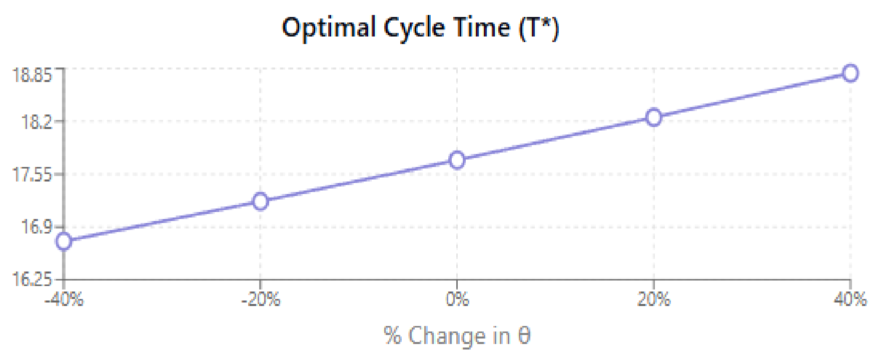

Figure 9 Optimal Cycle Time (T*)

Figure 9 illustrates the positive linear relationship between parameter θ changes (x-axis, -40% to +40%) and optimal cycle time T* (y-axis, 16.25-18.85 units). As θ increases, T* consistently rises, suggesting that higher values of this parameter necessitate longer cycle times for optimal operation. The effect appears relatively significant, with approximately a 2.6-unit change in T* across the full range of θ variation.

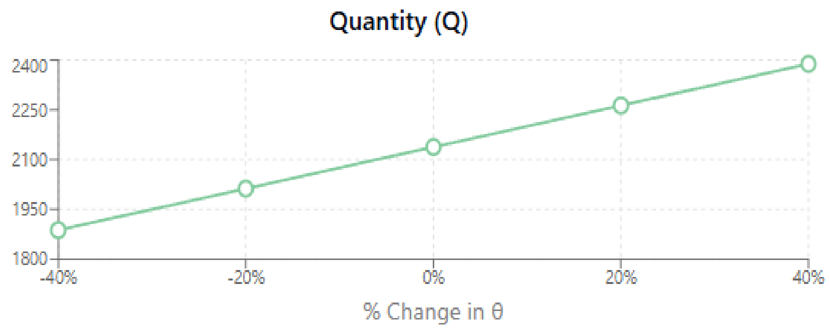

Figure 10 Quantity (Q)

Figure 10 shows the positive linear relationship between parameter θ changes (x-axis, -40% to +40%) and quantity Q (y-axis, 1800-2400 units). As θ increases, the order quantity consistently rises, with approximately a 600-unit increase across the full range. This suggests that higher values of θ require larger order quantities to maintain optimal inventory management, possibly due to changing demand or production conditions.

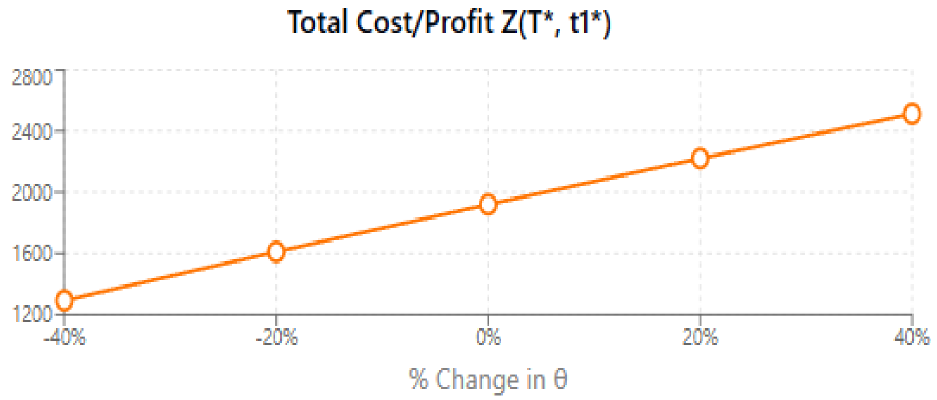

Figure 11 Total Cost/Profit (T*, t_1*)

Figure 11 displays the positive linear relationship between parameter θ changes (x-axis, -40% to +40%) and the total cost/profit function Z(T*, t1*) (y-axis, 1200-2500 units). As θ increases, the total cost/profit rises steadily, showing a substantial impact with approximately a 1300-unit increase across the full range. This indicates that higher θ values significantly increase system costs or decrease profitability.

8. OBSERVATIONS

The paper introduces a refined mathematical model to inventory control of products having finite lives and partial backlogging. It addresses a challenging issue in the present inventory system whereby multiple conflicting factors such as holding cost, shortage cost and deterioration cost are taken into consideration, under the time dependent demand. The author derives an overall model, more general than traditional inventory models, which takes into account both the decay of the product over time and the customer's patience to wait in

ISSN 2394 - 7780

the case of stock-out. It is, precisely for these reasons that this two-pronged approach to consider the decisions of production and inventory simultaneously, is apt for the industries where the products are perishable, and where the product fashion or technology imposes obsolescence. The model is developed using differential equations to account for depletion of inventory over initial (demand-only), demand plus deterioration, and shortage with partial backlogging periods. An advantage of the model is that it adopts clinically-based assumptions such as: time-varying linear demand ($D(t) = a + b_t$) and a general form of backlog function representing decreasing customer patience as waiting time in the system increases. These features are more realistic in capturing the real market mechanism by simple models with constant parameters. The solution method highlights the search for optimal values of cycle length (T) and time of initiation of shortage (t_1) by calculus-based optimization.

The model is illustrated numerically using a case study based on two dry food products (pasta and rice). The instance illustrates how a company can use the result to find optimal order quantity (2,137.66 order units), cycle length (17.72 days), and the strategic place to introduce shortage (15 days) to minimize total cost (₹1,920.43 per unit). This kind of practical implementation helps close the theoretical gap to real-world use. The sensitivity analysis yields valuable information about the impact of parameters on system performance. Of special importance is the result that the parameter of degradation (θ) and the mean lifetime (μ_0) becomes quite a dominant parameter in total cost with its effect on the objective function that is nearly close to double when θ rise of about 40%. This underscores the need to develop a reliable estimate of the rate of deterioration and product lifetime. On the other hand, the system exhibits more robustness against changes in demand parameter (b) and more sensitivity to backlogging parameter (α). The graphical presentation helps to visually understand these relationships and it can be seen how the various parameters affect the optimal cycle time, lot size and total cost. These diagrams facilitate more intuitive interpretation of the model in different situations, rendering more approachable the complicated mathematical relationships used by practitioners.

Practically, the model provides useful recommendations for inventory managers of items with short life spans. We develop an algebraic model for this with a view to supporting optimal decisions about stock levels by contributing to the understanding of trade-offs (in terms of holding costs, shortage penalties and deterioration losses). This can result in substantial enhancements to working capital turns, customer service levels and profit. The work is a contribution to the inventory management literature since it covers a difficult, but common, situation of many firm problems. With time-dependent demand, partial backlogging and deteriorating items, the proposed model is closer to the reality in inventory systems than the classic models with some unrealistic assumptions. The fully cost-based structure and optimization principle enable the firms to make more effective decisions regarding how to manage short-life-cycle products in the erratic market with any level of customers' backordering preferences.

9. CONCLUSION

The best replacement strategy for lifetime inventory with partial backlogging is an important breakthrough in inventory management both in theory and practice. By taking the recourse to strict mathematical modeling, this study has come up with a model too which properly accounts for the subtleties involved in products with finite life in a market with customers of diversified patience. The focus of the model is to combine time-dependent demand patterns, deterioration dynamics, and the realistic customer backlogging behavior into a single, but complete, approach that links theoretical elegance with practical applicability. The results from sensitivity analysis yield important implications for inventory managers, such as the uneven influence of the deterioration and lifetime parameters over total cost, while the changes of demand growth and backlogging parameters have a smaller effect. This indicates that it is essential to correctly estimate the deterioration rate and the life of products when making the inventory plan.

The pragmatic relevance of considering the (s, Q, CS) policy is spelt out in the case study in dry food products, which provides empirical support to the model that strategic choices regarding order quantities, cycle lengths, and planned shortages can notably reduce costs while not sacrificing service levels. The finding that the timing of replenishments continues to be effectively timed at some point across the parameter space provides useful operational insight since it indicates that depending on the value of the additional cost, firms can in fact coordinate some of the attributes of a replenishment plan even when other factors in the replenishment consideration set are subject to change. This study also provides insights and tools for academics and practitioners in the fields of inventory management; the tactics developed are based on yesterday s and today s best practices for multi-echelon systems where demand and lead times change. For future research, this model provides the groundwork for exploring the class of adaptive inventory policies that can change their structure due to changes in the market environment throughout the product life cycle. With the growing emphasis on sustainable prospects and resource efficiency, the adoption of these advanced inventory models is indispensable in minimizing waste as well as sustaining a competitive edge. By allowing more accurate calibration of inventory levels taking into account remaining product lifetime and customer behavior patterns, such an approach does not only improve financial performance but also (and possibly more importantly) is part of broader environmental and social objectives by minimizing waste due to obsolescence and improving customer satisfaction due to more reliable product availability.

REFERENCES

1 Wee, H. M. (1993). Economic production lot size model for deteriorating items with partial back-ordering. *Computers & Industrial Engineering, 24*, 449–458.

2 Sana, S. S. (2010). Optimal selling price and lot size with time-varying deterioration and partial backlogging. *Applied Mathematics and Computation, 217*, 185–194.

3 Sarkar, B. (2012). An EOQ model with delay in payments and time-varying demand. *Mathematical and Computer Modelling, 55*, 367–377.

4 Sett, B. K., Sarkar, B., & Goswami, A. (2012). A two-warehouse inventory model with increasing demand and time varying deterioration. *Scientia Iranica, 19*, 1969–1977.

5 Sarkar, B., Saren, S., & Cárdenas-Barrón, L. E. (2015). An inventory model with trade-credit policy and variable deterioration for fixed lifetime products. *Annals of Operations Research, 229*, 677–702.

6 Wu, J., Ouyang, L. Y., Cárdenas-Barrón, L. E., & Goyal, S. K. (2014). Optimal credit period and lot size for deteriorating items with expiration dates under two-level trade credit financing. *European Journal of Operational Research, 237*, 898–908.

7 Sarkar, B., Sarkar, S., & Yun, W. Y. (2016). Retailers optimal strategy for fixed lifetime products. *International Journal of Machine Learning and Cybernetics, 7*, 121–133.

8 Sarkar, B. (2016). Supply chain coordination with variable backorder, inspections, and discount policy for fixed lifetime products. *Journal of Industrial Engineering, Article ID 6318737*, 14 pages.

9 Ghare, P. M., & Schrader, G. F. (1963). A model for exponentially decaying inventory system. *International Journal of Production Research, 21*, 449–460.

10 Covert, R. P., & Philip, G. C. (1973). An EOQ model for items with Weibull distribution deterioration. *AIIE Transactions, 5*, 323–326.

11 Philip, G. C. (1974). A generalized EOQ model for items with Weibull distribution deterioration. *AIIE Transactions, 6*, 159–162.

12 Dave, U., & Patel, L. K. (1981). (T, Si) policy inventory model for deteriorating items with time proportional demand. *Journal of the Operational Research Society, 32*, 137–142.

13 Wee, H. M. (1997). A replenishment policy for items with a price-dependent demand and a varying rate of deteriorating. *Production Planning & Control, 8*, 494–499.

14 Goyal, S. K., & Giri, B. C. (2001). Recent trends in modeling of deteriorating inventory. *European Journal of Operational Research, 134*, 1–16.

15 Yang, P. C., & Wee, H. M. (2006). A collaborative inventory system with permissible delay in payment for deteriorating items. *Mathematical and Computer Modelling, 43*, 209–221.

16 Law, S. T., & Wee, H. M. (2006). An integrated production-inventory model for ameliorating and deteriorating items taking account of time discounting. *Mathematical and Computer Modelling, 43*, 673–685.

17 Chung, C. J., & Wee, H. M. (2008). An integrated production-inventory deteriorating model for pricing policy considering imperfect production, inspection planning and warranty-period and stock-level-dependent demand. *International Journal of Systems Science, 39*, 823–837.